Optimal Work Zone Lengths and Signal Timing for Two-Lane Highways Revisited: The Need to Consider Stochasticity

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ABSTRACT

Work zones can give rise to significant costs, both to maintenance agencies as well as to road users. In this paper we focus on the optimal configuration of work zones on two-lane highways. While previous work assumed determinism in the vehicle arrival processes, we relaxed this questionable assumption and allow vehicles to arrive according to some stochastic process. From the existing literature it is not clear whether such a relaxation would lead to different optimal work zone configurations, as compared to when determinism in the vehicle arrival process is assumed. In this paper we demonstrate that work zone configurations based on deterministic queuing theory is no longer optimal in a stochastic environment. Moreover, with the advent of computational power, we propose a practical, computationally feasible simulation optimization procedure to determine optimal work zone configurations. In a numerical case study we show that significant cost savings can be realized, especially when there are multiple work zones and/ or when work zone setup times are long.

INTRODUCTION

Work zones can give rise to significant costs, both to maintenance agencies as well as to road users. Hence it is critical to determine optimal policies regarding the location and configuration of work zones. The optimal location of work zones has been extensively studied in the literature. Following the classification in Ng et al. *(1)* we distinguish between short-term and long-term models. In short-term models, a list of sites in need of maintenance is assumed to be given, from which a subset is to be selected for immediate maintenance such that user delay is minimized (*2, 3, 4*). On the other hand, in long-term maintenance planning models, the location of the work zones are endogenous to the model and need to be determined based on predicted future conditions, e.g., see Gao and Zhang *(5)* and Ng et al. *(1, 6)*.

In this paper we assume that the work zone location is given and focus on the normative problem of optimal work zone configuration. For work zone studies of a more descriptive nature, we refer the reader to elsewhere in the literature *(7, 8, 9, 10)*. Various researchers have examined the optimal work zone configuration problem in the past *(11, 12, 13)*. For example, Memmott and Dudek *(14)* investigated the impacts of lane closures on a four-lane freeway section. A software tool based on deterministic queuing theory (QUEWZ) was developed to evaluate user cost due to work zones. Schonfeld and Chien *(15)* examined the optimal work zone lengths for two-lane highways. Analytical expressions were developed to determine the optimal work zone lengths, assuming uniform, deterministic vehicle arrivals. Chen and Schonfeld *(16)* relaxed this last assumption and allow for deterministic, time-dependent vehicle arrivals.

While previous research has addressed critical issues in the optimal configuration of work zones, they are all based on the questionable assumption of determinism. That is, vehicle arrivals were assumed to follow some deterministic queuing process, while in reality, arrivals of vehicles would be random in nature. This observation was also made in Chien et al. *(17)* in which microsimulation was used to evaluate delay at freeway work zones. They found that deterministic queuing theory tends to underestimate the actual delay and proposed an approximate method to relieve the computational burden of microsimulation in the delay estimation (note that while this might have been necessary at the time of that paper, with the increase and availability of computational power today, computer simulation has become a viable way to address real-life problems). The main contribution of this paper is to introduce stochasticity in the vehicle arrival processes *and* examine its impact on the optimal configuration (i.e. work zone length and signal timing) of work zones on two-lane highways. Note that Chien et al. *(17) only* examined the impact of stochastic vehicle arrivals on the calculated delay. *No attempt was made to find optimal work zone configurations assuming stochastic vehicle arrivals*. With the increase in computational power, we demonstrate that it has become feasible to determine optimal work zone configurations via simulation optimization *(18)*.

REVIEW OF THE TWO-LANE HIGHWAY WORK ZONE CONFIGURATION PROBLEM

As indicated in the previous section, the main focus of this paper is to investigate the impact of stochasticity on the configuration of work zones. In order to do so, we consider the special case of work zones on two-lane highways. However, we want to note that other highways (e.g., four lane highways) could have been used as well since the presented ideas and methodologies are general enough to encompass these cases.

Work zones on two-lane highways have been extensively studied in Schonfeld and Chien *(15)*, which forms the starting point for improvement in the current work. Work zones on two-lane highways often require the closure of one of the lanes. The open lane serves traffic from both directions alternately, while maintenance work is performed on the closed lane. When traffic from direction 1 is given right-of-way to the open lane, vehicles arriving from direction 2 wait and queue up and vice versa. Right-of-way is transferred to the other direction after some green and clearance time (see Figure 1). Figure 2 shows a typical space-time diagram associated with the two-lane work zone scenario. From approach 1, vehicles arrive (dotted arrows) during its green time g_1 and go through the work zone (of length L) without interruption. (Note that in the figure we have suggested that inter-arrival times between the vehicles are not uniform, but random.) The vehicles arriving from approach 1 after the green time are stopped and form a queue, waiting till their next green phase. A similar discussion can be held for approach 2 (solid arrows): vehicles arriving from approach 2 during g_1 and the following clearance time r form a queue, wait for their green time g_2 before they can go through the work zone; vehicles arriving during the clearance time of approach 2 are stopped and wait till their next green.

The length of work zones affects both the delay cost to travelers as well as the maintenance cost to the transportation agency. Longer work zones result in less repeated setups of equipment and thus reduce the fixed cost of maintenance. However, longer work zones necessitate longer cycle times that lead to greater queuing delay to users. User delay is further exacerbated because of the reduced speeds in work zones. Hence it is clear that user delay depends on both the green time allocated to each of the approaches as well as the length of the work zone. The objective in designing two-lane work zone configurations is to find optimal work zone lengths and cycle times which minimize the total cost, defined as the sum of user delay cost and maintenance cost.

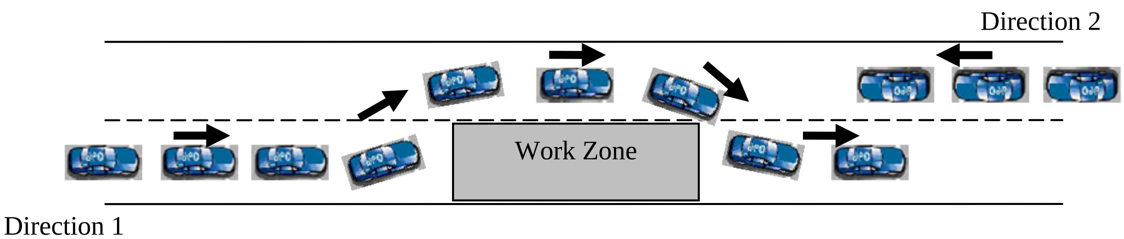

FIGURE 1 Illustration of a two-lane work zone.

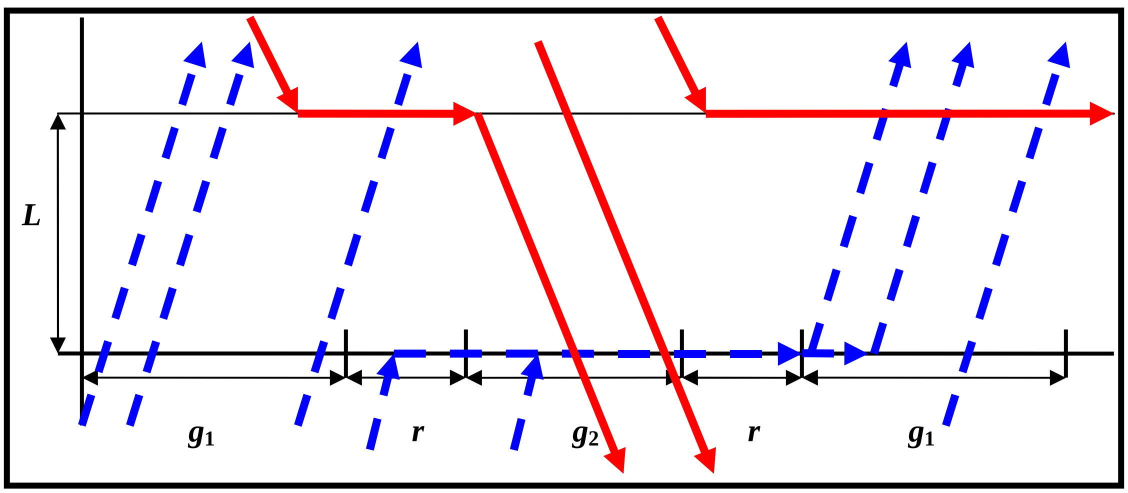

FIGURE 2 Typical space-time diagram two-lane work zone scenario.

Schonfeld and Chien *(15)* showed that when vehicle arrival processes are deterministic and uniform, the optimal work zone length L^* (in miles) and the optimal green times (g_1^*, g_2^*) – in seconds – are given by (without loss of generality, we index the two approaches by 1 and 2):

$$L^* = \sqrt{\frac{z_1 V\left(\frac{3600}{H} - Q_1 - Q_2\right)}{z_4\left[Q_1\left(\frac{3600}{H} - Q_1\right) + Q_2\left(\frac{3600}{H} - Q_2\right)\right] v}} \tag{1}$$

$$g_1^* = \frac{r^*\left(\frac{3600}{H} + Q_1 - Q_2\right)}{\left(\frac{3600}{H} - Q_1 - Q_2\right)} - r^* \tag{2}$$

$$g_2^* = \frac{r^*\left(\frac{3600}{H} + Q_2 - Q_2\right)}{\left(\frac{3600}{H} - Q_1 - Q_2\right)} - r^* \tag{3}$$

$$r^* = \frac{L^*}{V} \tag{4}$$

where

z_1 = fixed cost for setting up a work zone (in \$);

z_4 = additional setup time required per work-zone mile (in hours/mile);

Q_1 = traffic flow from direction 1 (in vehicles per hour, vph);

Q_2 = traffic flow from direction 2 (in vehicles per hour, vph);

V = average speed in a work zone (in mph);

v = average user delay cost (in \$/veh-h);

H = headway of the vehicles (in seconds);

r^* = optimal clearance time (in seconds).

The total cost C_T can then be calculated as

$$C_T = C_U + C_m \tag{5}$$

where

$$C_U = \frac{Y\left(\frac{z_3}{L^*} + z_4\right)v}{g_1^* + g_2^*} \tag{6}$$

$$C_m = \left(\frac{z_1}{L^*} + z_2\right) \tag{7}$$

and

C_U = user delay cost per maintained mile (in \$);

C_m = average maintenance cost per mile (in \$);

z_2 = average additional maintenance cost per work zone mile (in \$/ mile);

z_3 = setup time of the work zone (in hours);

Y = incurred user delay in one cycle (in veh-h).

For more details on the derivation of the above formulas, we refer the reader to the original paper *(15)*.

THE NEED TO ACCOUNT FOR STOCHASTICITY

It is without doubt that the assumption of deterministic vehicle arrivals is questionable in practice *(17)*. However, it is *not* clear whether such an assumption also leads to different optimal work zone configurations. In other words, optimal configurations obtained assuming determinism, might remain optimal in a stochastic world. In this section we provide an example to demonstrate that this generally does not hold. In the next section we present a systematic simulation optimization procedure that can be employed to find optimal work zone configurations for two-lane highways under stochastic vehicle arrivals. The results demonstrate that significant cost savings can be realized using the proposed method.

Consider the following situation. On a two-lane highway work zone, suppose that vehicles arrive at the approaches at a rate of $Q_1 = Q_2 =$730 vph. The headways H are assumed to be 2 seconds; the work zone speed V is 40 mph and the user delay cost v is assumed to be \$25/vehicle-hour, following Huen et al. *(19)*. The average maintenance cost (z_2), additional work zone setup time (z_4), the fixed setup cost (z_1) and work zone setup time (z_3), are set to be 50,000 \$/mile, 9.7 h/mile, 1,000 \$/zone and 5 h/mile, respectively. By using equations (1)-(4), the optimal work zone length and green times are calculated as 0.3 mile and 120 seconds, respectively.

To account for stochastic vehicle arrivals, we assume that vehicles arrive according to a Poisson process at both of the approaches (with the same mean as above, i.e. 730 vph). A Poisson process is known to capture vehicle arrivals fairly accurately in relative light traffic conditions, e.g. see page 44 of *(21)*, which we hence conjecture is appropriate to model vehicle arrivals at two-lane rural highways. Since we rely on simulation, clearly, more general renewal processes can be used to describe the vehicle arrivals processes. For illustration purposes, we simply adopt the Poisson process. We used CORSIM (a microsimulation package developed by the Federal Highway Administration) to evaluate the traffic delay Y in equation (6) when vehicle arrivals follow a Poisson process.

To ensure a fair comparison with the deterministic case, we adopted to the extent possible the same parameters as described above, e.g. the speed limit at the work zone was set to be 40 mph. The second column in Table 1 (called solution 1 in the table) shows the various costs obtained from simulation for the "optimal" work zone configuration determined by equations (1)-(4), but assuming stochastic vehicle arrivals. For example, we see that the *simulated* total cost C_T equals $69,587 for solution 1 (this and subsequent values were obtained as the average of a sufficiently large number of simulation runs, each simulating one hour of operation). With a simple modification (the green times are reduced from 120 seconds to 100 seconds), one can already obtain a solution whose total cost ($69,044) is lower than the total cost of solution 1 (see solution 2 in Table 1). By changing both the work zone length *and* green times, an even better solution ($C_T = \$65{,}762$) can be obtained (solution 3 in Table 1). We want to remark that we did not perform any systematic optimization (as will be done in the next section) to arrive at solutions 2 and 3 (resulting in cost savings that might seem to be modest) since our primary goal here was to demonstrate that work zone configurations obtained based on determinism, are no longer optimal in real-world, stochastic environments. Clearly, our goal is achieved.

TABLE 1 Comparison of Work Zone Configurations

	Solution 1 (assuming determinism): $L^* = 0.3$ mile $g_1^*, g_2^* = 120$ seconds	Solution 2: $L^* = 0.3$ mile $g_1^*, g_2^* = 100$ seconds	Solution 3: $L^* = 0.5$ mile $g_1^*, g_2^* = 100$ seconds
C_U	$16,306	$15,762	$13,762
C_m	$53,282	$53,282	$52,000
C_T	$69,587	$69,044	$65,762

NUMERICAL CASE STUDY: SIMULATION OPTIMIZATION

In this section we demonstrate how simulation optimization can be used to efficiently determine optimal work zone length and green time combinations. The feasibility and effectiveness of the proposed approach relies on the following two important observations. First, microsimulation has become a trustworthy and effective tool in the evaluation of "what if" scenarios. While the simulation of entire metropolitan networks remains a computational challenge on a microscopic scale, computing power has increased dramatically in recent years to be able to easily simulate traffic at two-lane highways. As in *(15)* we do not assume that traffic diverts to other parts of the network; others have examined this possibility, e.g. Chen and Schonfeld *(16)* and Ng et al. *(1)*. Second, for a given work zone location – assuming that the mean number of vehicle arrivals is known– only a *limited* number of triplets (L, g_1, g_2) need to be considered in practice. For example, it is impractical and even useless to determine work zone lengths at the resolution of inches. More likely, one would like to consider increments in work zone lengths of the order of 0.05 mile. As for the green times, increments of 10 seconds would be appropriate. Moreover, there are natural lower and upper bounds on these variables. For instance, engineering judgment would probably tell us that a 20-mile work zone would not be very practical. Henceforth, we shall refer to combinations of (L, g_1, g_2) that satisfy the above "engineering-judgment-conditions" as feasible.

Given the above two observations, we propose to use the following (computationally feasible) simulation optimization procedure. Let *V'*, *FL* and *FG* denote the speed limit, feasible set of work zone lengths L and feasible set of green times (g_1, g_2), respectively. Then the algorithm can be stated as:

<u>Algorithm Optimal Work Zone Configuration by Simulation Optimization</u>

INPUT: $z_1, z_2, z_3, z_4, V', Q_1, Q_2\ v, FL, FG$
OUTPUT: optimal work zone length and green times

FOR each work zone length *L* in *FL*
DO
Evaluate the total cost C_T using microsimulation for each combination (g_1, g_2) that is in *FG*
END DO
END FOR

SELECT among all combinations (*L*, g_1, g_2) the triple that yields the lowest cost C_T

Now let us demonstrate the above simulation optimization algorithm based on the work zone data used in the previous section of this paper (see second paragraph of previous section). In order to find the optimal combination of the triplet (*L*, g_1, g_2), we first consider the case when *L* = 0.1 mile. For this value of *L*, we perform simulation runs in CORSIM to evaluate user and maintenance costs for different values of the green times (with increments of 10 sec). Among these cases, the combination of green times (g_1, g_2) that yields the minimum total cost C_T is designated as the "optimal" green times associated with *L* = 0.1 mile. Then, we change the work zone length with increments of 0.05 mile and repeat the process above. Table 2 lists selected simulation results (for conciseness we have not shown all simulation results). For a given combination (*L*, g_1, g_2), each run of the CORSIM-simulation (as above, we simulated one hour as a representative time period) consumes *less than 1 second* computational time on a standard desktop computer (with 1 GB of memory and a 3.4 GHz CPU). Several runs were performed for the same (*L*, g_1, g_2) combination and the average values of these runs are presented in Table 2. Based on the presented results, one can see that the optimal work zone length and green times were found to be equal to 0.6 mile and 110 seconds, respectively (simulation No. 12 in Table 2). The corresponding total cost C_T is $64,406, which is 7.5% less than the cost (simulation No. 3 in Table 2) associated with the configuration obtained using equations (1)-(4). Note that this cost saving is only for one *single* work zone. In a given year, there are typically a whole lot more roads to be maintained. That is, our results indicate that by relaxing the assumption of determinism, one could potentially realize substantial monetary savings.

Finally, we want to make the following observation regarding the work zone setup time. One can verify that the optimal work zone configuration remains the same (i.e. simulation No. 12 in Table 2 remains optimal) as long as z_3 is shorter than 30 hours. The difference in C_T between this solution and the deterministic solution (simulation No. 3 in Table 2), however, increases dramatically with z_3 as illustrated in see Figure 3. In other words, we see that it is particularly critical to consider stochasticity when the setup times of work zones are long.

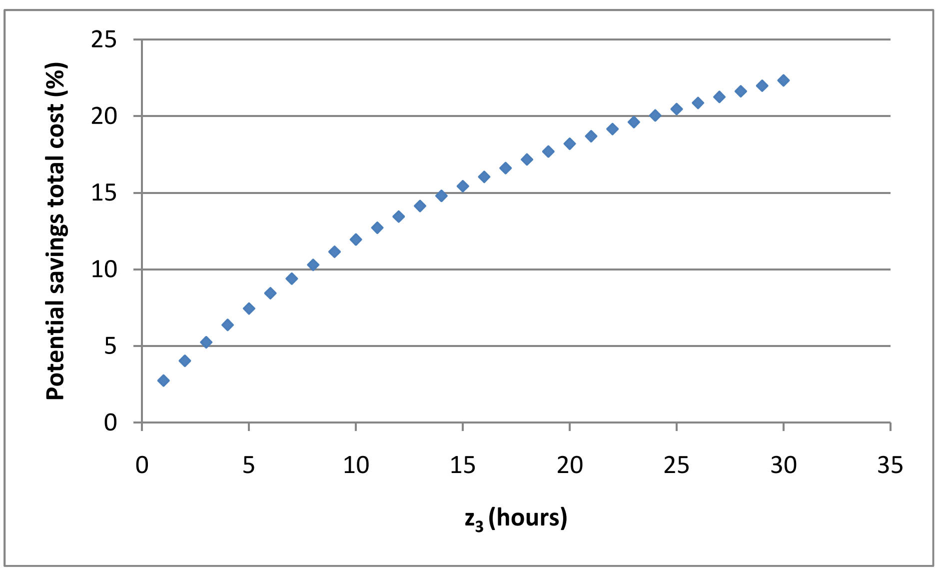

FIGURE 3 Potential cost savings versus the work zone setup time z_3 by accounting for stochasticity.

TABLE 2 Selected Simulation Optimization Results

Simulation No.	C_T	C_U	C_m	L	g_1, g_2
1	$72,884	$12,874	$60,010	0.1	70
2	$68,286	$13,236	$55,050	0.2	80
3	**$69,587**	**$16,306**	**$53,282**	**0.3**	**120**
4	$69,044	$15,762	$53,282	0.3	100
5	$67,203	$14,703	$52,500	0.4	100
6	$65,762	$13,762	$52,000	0.5	100
7	$65,525	$13,707	$51,818	0.55	100
8	$64,451	$12,784	$51,667	0.6	100
9	$66,853	$15,186	$51,667	0.6	80
10	$69,039	$17,373	$51,667	0.6	70
11	$66,610	$14,943	$51,667	0.6	120
12	**$64,406**	**$12,739**	**$51,667**	**0.6**	**110**
13	$65,243	$13,576	$51,667	0.6	105
14	$65,030	$13,491	$51,538	0.65	100
15	$64,968	$13,539	$51,429	0.7	110
16	$65,056	$13,627	$51,429	0.7	100
17	$65,152	$13,901	$51,251	0.8	90
18	$65,423	$14,172	$51,251	0.8	100
19	$65,532	$14,281	$51,251	0.8	110
20	$65,522	$14,344	$51,178	0.85	100
21	$65,130	$14,018	$51,112	0.9	90
22	$65,374	$14,262	$51,112	0.9	100
23	$66,041	$14,929	$51,112	0.9	110
24	$66,110	$14,998	$51,112	1	90
25	$66,012	$15,012	$51,000	1	100
26	$65,975	$14,975	$51,000	1	80

CONCLUSIONS

Road maintenance is essential to ensure a safe and efficient transportation system. In this paper we focused on the optimal configuration of work zones on two-lane highways. Unlike previous work, we relaxed the assumption of determinism and allowed vehicle arrivals to be stochastic, which is arguably more realistic. While previous work has shown that this relaxation is important in the quantification of user delay at work zones, it is not trivial that stochasticity would lead to different optimal work zone configurations. That is, optimal configurations assuming determinism might remain optimal in a stochastic environment. In this paper we provided examples to show that this is not the case. In other words, it is critical to consider stochasticity in the work zone configuration problem.

To determine optimal work zone configurations under stochastic vehicle arrivals, we proposed a simulation optimization algorithm. The algorithm was shown to be computationally feasible because of the limited number of possible work zone configurations in practice.

In a numerical case study we demonstrated that the "optimal" work zone configurations obtained based on deterministic queuing theory, are no longer optimal when vehicle arrivals are allowed to be random. Moreover, we demonstrated that the cost savings resulting from a simulation optimization approach can be substantial, especially when one considers multiple work zones and/ or when work zone setup times are long.

In this paper we examined work zones on two-lane highways to illustrate the need to consider stochasticity. It is clear that the proposed simulation optimization approach is applicable to more general highway configurations. In future work we intend to investigate the potential savings in cost for these more general highway classes. Moreover, we will also examine the impact of the stochastic vehicle arrival process on the cost savings (i.e. the Poisson assumption will be relaxed). Finally, we will aim to obtain analytical solutions using (stochastic) queuing theory. In fact, similar work has already been reported in the literature. For example, Van der Heijden et al. *(20)* examined an underground transportation system in which Automated Guided Vehicles carry cargo through shared underground tunnels. The problem displays remarkable similarities with the work zone configuration problem described in this paper. However, their model implicitly assumed the presence of advanced detection systems that will not be present at temporary work zones. Therefore, while accounting for the random nature of vehicle arrivals, their model cannot be directly applied in the context of work zones. More accurate stochastic models need to be built, tailored to the work zone problem.

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